

Method: T-test analysis to determine adequacy of sample set when fewer than 30 samples

Based on method in Statistics for Dummies, 2003.

Problem: 10 SPT blow counts taken in CH, some CL and MH, shallow depth, no GWT

SPT Blow Count Results

	27 blows/ft
	10 blows/ft
	13 blows/ft
	13 blows/ft
	16 blows/ft
	6 blows/ft
	10 blows/ft
	14 blows/ft
	8 blows/ft
	13 blows/ft
Average	13 blows/ft

Given that only 10 samples were taken, how do we know how close our computed sample average is to the true population average?

Solution: Apply t-test to compute a Confidence Interval.

Let's say we are shooting for a 95% Confidence Interval of what an average strength by each method is.

So we have computed averages above, but since based on only 10 samples, in what range of values could the average be to be in the 95% CI?

P. 206 of Statistics for Dummies, 2003:

1. Determine the confidence level and find the appropriate Z-value. (See Table 10-1 of text, p.180.)

% Confidence	Z-Value
80	1.28
90	1.64
95	1.96
98	2.33
99	2.58

Z = 1.96

2. Find the sample mean, sample std deviation, and sample size.

xbar	13
s	8
n	10
CoV	0.59

3. Multiply Z * s and divide by square root of n to compute margin of error.

Margin of Error = 5

4. Take xbar plus or minus the Margin of Error to obtain the Confidence Interval.

However, when your sample size is small (less than 30), a modification is needed. (See Chapter 15.)

P. 238 of text:

1. Compute xbar, s, and n. Done above.

2. Find xbar minus uo (uo being the true population average, which we don't know, so will assume).

The way this will work, just assume an average lower than the computed. We will then adjust it until we get it right at 95% CI.

Assume uo	7.5 blows/ft
xbar - uo	5.5 blows/ft

3. Calculate standard error s divided by sqrt(n).

Standard Error = 2

4. Divide Step 2 result by Step 3 SE.

Test statistic = 2.28

This result is the test statistic.

This means the sample mean (computed average) is 2.28 standard errors above/below the assumed/claimed population mean of 7.5

Decide if this test statistic supports our assumption/claim that the population mean is 7.5, calculate p-value.

However, since sample size is small, look up test statistic on the t-distribution and text in Chapter 14.

Per P. 220 on:

5. Make a null hypothesis H_0 that the assumed population mean of 7.5 is correct.

Alternative hypothesis H_a is that population mean is not equal to H_0 (as opposed to $<$ or $>$).

Again, we will use a 95% confidence interval to determine if null hypothesis is true or not.

6. Because degrees of freedom = sample size - 1:

DoF 9

7. Use Table 14-2 (p. 233).

2.28 falls on 97.5th % for 9 DoF.

This is lower-bound for 2 tails, as % difference between 100 and 97.5 doubled is 5% corresponding to 95% CI.

8. So correctly rounded, answer is: 8 blows/ft

Table 8-1

Standard Score	Percentile
-3.4	0.03%
-3.3	0.05%
-3.2	0.07%
-3.1	0.10%
-3	0.13%
-2.9	0.19%
-2.8	0.26%
-2.7	0.35%
-2.6	0.47%
-2.5	0.62%
-2.4	0.82%
-2.3	1.07%
-2.2	1.39%
-2.1	1.79%
-2	2.27%
-1.9	2.87%
-1.8	3.59%
-1.7	4.46%
-1.6	5.48%
-1.5	6.68%
-1.4	8.08%
-1.3	9.68%
-1.2	11.51%
-1.1	13.57%
-1	15.87%
-0.9	18.41%
-0.8	21.19%
-0.7	24.20%
-0.6	27.42%
-0.5	30.85%
-0.4	34.46%
-0.3	38.21%
-0.2	42.07%
-0.1	46.02%
0	50.00%
0.1	53.98%
0.2	57.93%
0.3	61.79%
0.4	65.54%
0.5	69.15%
0.6	72.58%
0.7	75.80%
0.8	78.81%
0.9	81.59%
1	84.13%
1.1	86.43%
1.2	88.49%
1.3	90.32%
1.4	91.92%
1.5	93.32%
1.6	94.52%
1.7	95.54%
1.8	96.41%
1.9	97.13%
2	97.73%
2.1	98.21%
2.2	98.61%
2.3	98.93%
2.4	99.18%
2.5	99.38%
2.6	99.53%
2.7	99.65%
2.8	99.74%
2.9	99.81%
3	99.87%
3.1	99.90%
3.2	99.93%
3.3	99.95%
3.4	99.97%

Table 14-2

DoF	90th %ile	95th %ile	97.5th %ile	98th %ile	99th %ile
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.92	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.44	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.86	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.25
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.35	1.771	2.16	2.65	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.12	2.583	2.921
17	1.333	1.74	2.11	2.567	2.898
18	1.33	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.08	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.5	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.06	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.31	1.697	2.042	2.457	2.75
40	1.303	1.684	2.021	2.423	2.704
60	1.296	1.671	2	2.39	2.66
Z-values	1.282	1.645	1.96	2.326	2.576

Method: Obtaining Confidence Interval Neglecting Small Sample Size

Based on method in Statistics for Dummies, 2003.

Problem: 10 SPT blow counts taken in CH, some CL and MH, shallow depth, no GWT

SPT Blow Count Results

27 blows/ft
10 blows/ft
13 blows/ft
13 blows/ft
16 blows/ft
6 blows/ft
10 blows/ft
14 blows/ft
8 blows/ft
13 blows/ft

Average 16 blows/ft

Solution: Find confidence interval neglecting small sample size.

Let's say we are shooting for a 95% Confidence Interval of what an average strength by each method is.

So we have computed averages above, but since based on only 10 samples, in what range of values could the average be to be in the 95% CI?

P. 206 of Statistics for Dummies, 2003:

1. Determine the confidence level and find the appropriate Z-value. (See Table 10-1 of text, p.180.)

Z 1.96

% Confider Z-Value	
80	1.28
90	1.64
95	1.96
98	2.33
99	2.58

2. Find the sample mean, sample std deviation, and sample size.

xbar 16
s 8
n 10
CoV 0.48

3. Multiply Z * s and divide by square root of n to compute margin of error.

Margin of Error 5

4. Take xbar plus or minus the Margin of Error to obtain the Confidence Interval.

CI 11 to 20

5. Use lower bound and round to correct significant figures.

CI-lower bound 10 blows/ft
9 blows/ft

Method: ASTM E122 Methodology

Problem: 10 SPT blow counts taken in CH, some CL and MH, shallow depth, no GWT

SPT Blow Count Results

	27 blows/ft
	10 blows/ft
	13 blows/ft
	13 blows/ft
	16 blows/ft
	6 blows/ft
	10 blows/ft
	14 blows/ft
	8 blows/ft
	13 blows/ft
Average (\bar{x})	13 blows/ft
Std Dev (s)	6 blows/ft
n	10

E from Equation 1 of ASTM E122

E (precision) 5 blows/ft

So best you can say is average is between 18 and +/- 8 blows/ft

Rounding with correct significant digits, SPT blow count that should be used in computations is:

8 blows/ft non-rounded

8 blows/ft Final Answer

Formula in ASTM E122 accounts for 3 standard deviations. For 95% confidence, should we use just 2 standard deviations?

E (precision) 4 blows/ft

So best you can say is average is between 17 and +/- 9 psf

Rounding with correct significant digits, strength value that should be used in computations is:

9 blows/ft non-rounded

9 blows/ft Final Answer

Method: 2 Standard Deviations from Mean captures 95% of all possibilities based on Empirical Rule

Problem: 10 SPT blow counts taken in CH, some CL and MH, shallow depth, no GWT

SPT Blow Count Results	
	27 blows/ft
	10 blows/ft
	13 blows/ft
	13 blows/ft
	16 blows/ft
	6 blows/ft
	10 blows/ft
	14 blows/ft
	8 blows/ft
	13 blows/ft
Average (\bar{x})	13 blows/ft
Std Dev (s)	6 blows/ft
n	10

Average less 2 std deviations = 1 blows/ft

Rounding to significant figures = 1 blows/ft

Interesting question: Should any of the high/low data be thrown out?

First, check to be sure there was no mistake made in collecting the extreme data (27 on the high end, 6 and 8 on the low end). One method to use is Chauvenet's Criterion:

1. Calculate difference between sample mean and suspect data, and how many standard deviations that quantity is.

Data point	# StDevs from Mean
27	2.4
6	1.2
8	0.9

2. Use a Normal Error Integral Table (published tables as in Taylor 1982) to look up the probability that a measurement will differ from the mean by the computed number of standard deviations.

Data point	# StDevs from Mean	Probability Outside Computed # StDevs from Mean
27	2.4	0.016
6	1.2	0.23
8	0.9	0.368

3. Since we took 10 measurements, multiply probabilities by 10. If result is less than 1/2, consider rejecting the measurement.

Data point	# SDs Off	Probability	x10	Decision
27	2.4	0.016	0.16	Reject
6	1.2	0.23	2.3	Accept
8	0.9	0.368	3.68	Accept

4. If the data point of 27 is rejected, the sample mean and standard deviation should be recalculated based on the remaining 9 measurements.